Reply by Authors to B.T.F. Chung and L. T. Yeh

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THE paper[1] by the present authors was intended to obtain a closed-form solution for ablation of melting solids due to radiative heating. It provides results for all values of β, and reduces costly computer time in computation of numerical results in comparison with the results predicted from numerical analysis. Also, the analysis gives solutions that are restricted to those values of β for which they are computed.

The manuscript of this paper has been compared with the material in print. Some typographical errors, which were overlooked by the present authors, are found to exist. Accordingly, in Eq. (20), $60c\rho L$ is to be replaced by $(20/3)c\rho L$. This equation is same as Eq. (20) of the earlier paper[2] of one of the present authors. In Eq. (22), 40Φ is to be corrected to 120Φ, and in Eq. (24) a large parenthesis is to be incorporated on the left-hand side, whereas the second term within the small parentheses on this side is to be multiplied by Φ. As a result, it becomes the equation mentioned by Chung and Yeh in their Comment. The subsequent errors arise in the paper due to this error in Eq. (24), although Eqs. (25) and (27), respectively, are closed-form solutions for any values of β and $\beta=0$ of Eq. (24), and the numerical solution of Eq. (28) yields the same results as that of Eq. (25).

References

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Comment on "Turbulent Boundary Layer with Injection and Surface Roughness"

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IN a recent paper, Schetz and Nerney[1] have presented a valuable set of results for skin-friction coefficients and velocity profiles over rough, porous surfaces. Although the present writer agrees with their remarks that earlier measurements of skin friction with injection are inconsistent, he does not agree with the implication that they can be disregarded. Certainly a recent reanalysis[2,3] by the present author of the data of Simpson et al.[4] and McQuaid[5] has shown that there are genuine differences between the levels of skin friction measured in the two experiments. However, the analysis has also shown that with zero injection the skin-friction coefficients from both sets of experiments are unlikely to be more than 10% above the corresponding solid-surface values. Furthermore, it should be noted that Anderson[6] has recently repeated some of Simpson's tests using a new method to measure skin friction. This method measures shear-stress distributions through the layer using hot wires, and should be more accurate than the momentum-integral approach used by Simpson et al. All the skin-friction results obtained are slightly *lower* than those found by Simpson et al., and the skin-friction coefficients with zero injection are very close to the corresponding solid-surface values. Thus it appears that the earlier results do not show the large increases in skin friction above the solid-surface values as found by Schetz and Nerney. This is almost certainly due to differences in surface roughness. In particular, the electron-microscope photograph (Fig. 4 of Ref. 5) of the sintered polythene surface used by McQuaid appears to show a much smoother surface than that shown in the corresponding photograph in Ref. 1. It should also be noted that early tests on suction surfaces in the Engineering Department at Cambridge showed that it was very difficult to measure reliable boundary-layer developments over porous surfaces with zero injection. Essentially, it was found that small pressure variations over the surface caused small inflows and outflows locally, and these flows had a large effect on the boundary-layer development. In particular, small inflows (i.e., suction) could result in large increases in skin friction. In this connection, it would be interesting to investigate the effect of the gap around the skin-friction element on the measured skin-friction coefficient since even on solid surfaces the effect of gap size is not completely understood.

As is well known, the value of the skin-friction coefficient has a large effect on the level of the law of the wall and it is interesting to note that Schetz and Nerney found that all their profiles with injection could be plotted† as

$$\frac{u}{u_\tau} = A\log_{10}y^+ + B \tag{1}$$

whereas all earlier workers have found a law of the form

$$(2/v_0^+)[(1+v_0^+u^+)^{1/2}-1] = A\log_{10}y^+ + B(v_0^+) \tag{2}$$

Furthermore, they have all agreed that, in Eq. (2), the slope A is almost independent of injection, but there is strong disagreement over the variation of B with v_0^+ (see, for example, Fig. 18 of Ref. 3). For small values of v_0^+, Eq. (2) approaches Eq. (1), and it is difficult to distinguish between the two laws. However, for large v_0^+ (say >0.2), it is clear that Eq. (2) implies a nonlinear variation of u/u_τ with $\log y^+$. This is illustrated in Fig. 1, where a profile measured by Simpson for $\dot{m}/\rho U = 0.078$ is plotted as u/u_τ and as $(2/v_0^+)[(1+u^+v_0^+)^{1/2}-1]$. For this profile, Simpson et al. quote a value of c_f of 0.00028, and this value is close to that obtained by Squire in his reanalysis of the original data. However, to show the effects of changes in skin friction, results are also plotted for a fourfold increase ($c_f = 0.00112$) and a fourfold decrease ($c_f = 0.00007$) in skin friction. As will be seen, the plot of $(2/v^+)[(1+u^+v_0^+)^{1/2}-1]$ against $\log y^+$ has a clear linear portion with a slope that varies slightly ($5.0<A<5.5$), whereas the plot of u/u_τ against $\log y^+$ is nonlinear with a clear point of inflection; furthermore, the slope at the point of inflection varies widely with skin friction ($9.5<A<35$) and is always much greater than the flat plate valve ($A = 5.5$).

In conclusion, the present author believes that the main value of Ref. 1 is the demonstration of the importance of surface roughness for porous surfaces. Furthermore, this

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†Nomenclature as Ref. 1.

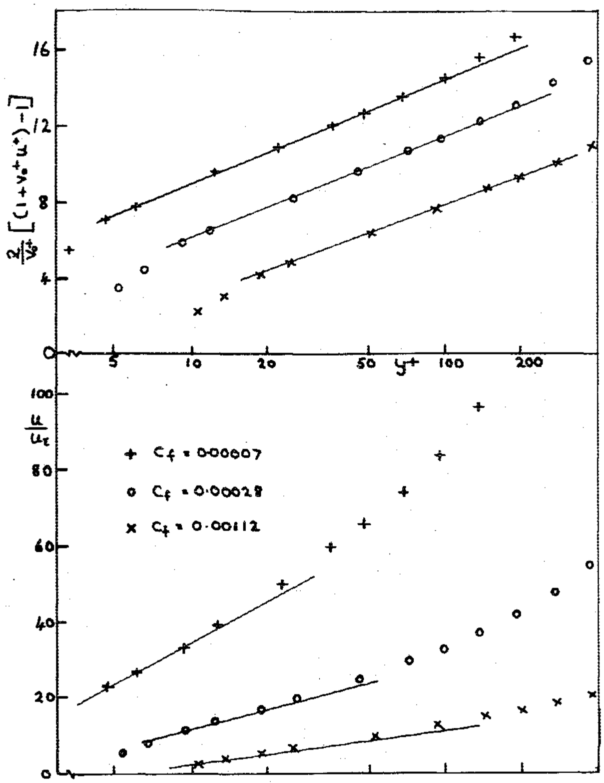

Fig. 1 Law of the wall.

factor may explain the inconsistencies noted in earlier work. Thus future workers must specify full details of the surfaces used. Also many more data are required before a definitive law-of-the wall can be established.

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Reply by Authors to L.C. Squire

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THE authors wish to thank Prof. Squire for his thoughtful comments on our work.[1] We should like to offer some brief observations in reply.

First, it is not our contention that all previous, indirectly determined wall friction data with injection should be completely disregarded. We do contend, however, that directly measured values are much to be preferred.

Second, the method of extrapolating shear values measured with a hot-wire out in the boundary layer to the wall has proved unreliable for solid wall flows in the past. We do not believe that this method offers a useful alternative to the direct measurement approach.

Third, we do hope that our work concerning roughness effects of porous surfaces will indeed cause future workers to carefully specify the surface under study. We agree that more data are needed and urge that surfaces made of practical porous materials receive attention.

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